Saturation problem

- Two most important/elusive numbers in nuclear physics

- Historical perspective
- Hole-line expansion
- Conclusions but no solution!

- GFMC for light nuclei
- Some considerations and observations…

- Assess original assumptions
- Personal perspective & some recent results with chiral NN & NNN

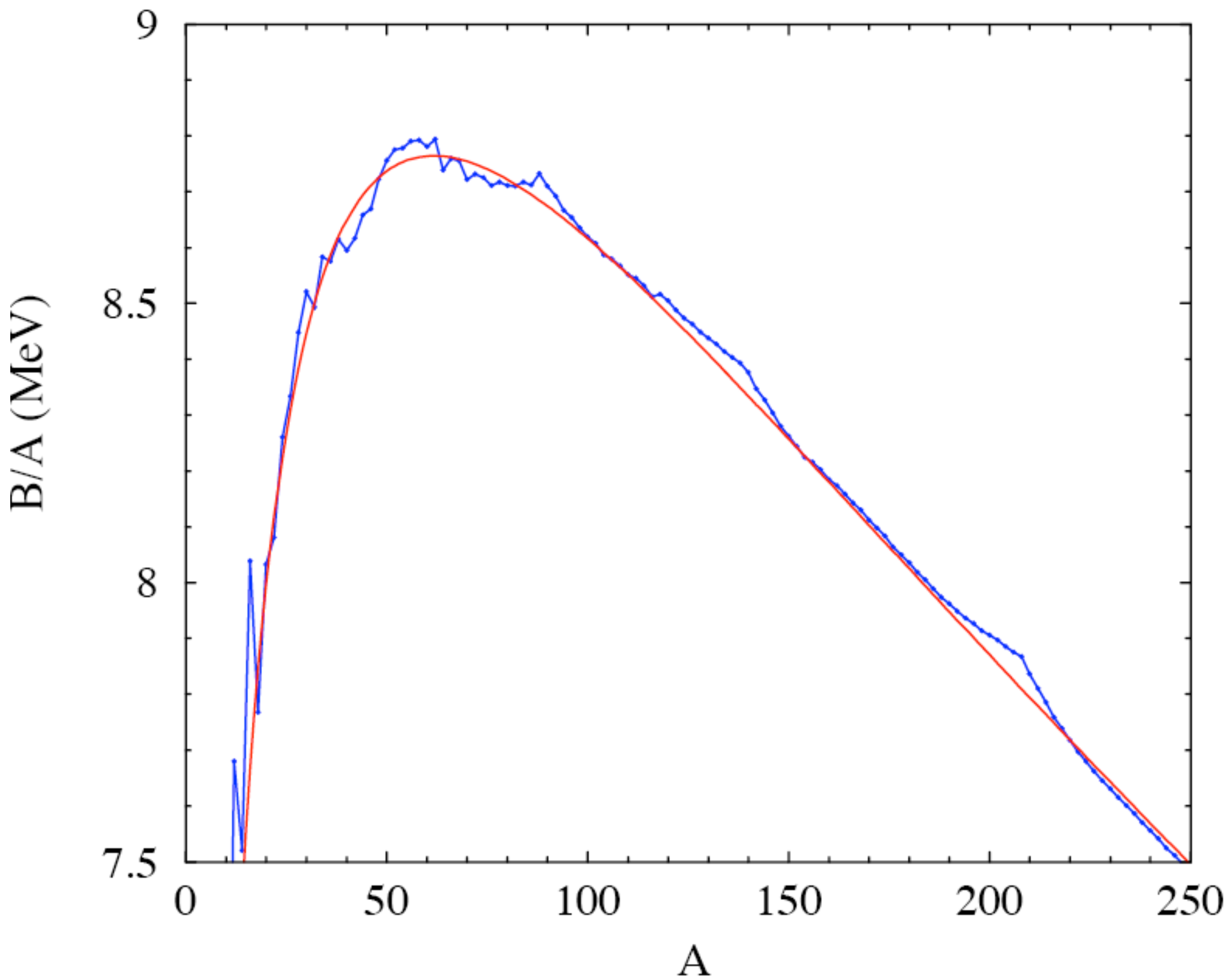
Empirical Mass Formula

Global representation of nuclear masses (Bohr & Mottelson)

$$B = b_{vol} A - b_{surf} A^{2/3} - \tfrac{1}{2} b_{sym} \frac{(N-Z)^2}{A} - \tfrac{3}{5} \frac{Z^2 e^2}{R_c}$$

- Volume term b_{vol} = 15.56 MeV
- Surface term b_{surf} = 17.23 MeV
- Symmetry energy b_{sym} = 46.57 MeV
- Coulomb energy R_c = 1.24 $A^{1/3}$ fm
- Pairing term must also be considered

Empirical Mass Formula



Plotted: most stable nucleus for a given *A*

Central density of nuclei

Multiply charge density at the origin by A/Z

$\Rightarrow$ Empirical density = 0.16 nucleons / fm^3

$\Rightarrow$ Equivalent to $k_F = 1.33\ fm^{-1}$

Nuclear Matter

N = Z

No Coulomb

$A \Rightarrow \infty$, $V \Rightarrow \infty$ but $A/V = \rho$ fixed

"Two most important numbers"

b_{vol} = 15.56 MeV and $k_F = 1.33\ fm^{-1}$

Historical Perspective

- First attempt using scattering in the medium — *Brueckner* 1954
- Formal development (linked cluster expansion) — *Goldstone* 1956
- Low-density expansion — *Galitskii* 1958
- Reorganized perturbation expansion (60s) — *Bethe & students*

  involving ordering in the number of hole lines — *BBG*-expansion
- Variational Theory vs. Lowest Order *BBG* (70s) — *Clark, Pandharipande*
- Variational results & next hole-line terms (80s) — *Day, Wiringa*
- Three-body forces? Relativity? (80s) — *Urbana, CUNY*
- Confirmation of three hole-line results (90s) — *Baldo et al.*
- New insights from experiment — *NIKHEF, Amsterdam*

  about what nucleons are up to in the nucleus (90s & 00s) — *JLab, Newport News, VA*
- *ONGOING to this day... with more emphasis on asymmetric matter ... symmetry energy*

Saturation properties of nuclear matter

- Colorful and continuing story
- Initiated by Brueckner: proper treatment of SRC in medium -> ladder diagrams but only include pp propagation

$$\langle \boldsymbol{k} m_\alpha m_{\alpha'} | G(\boldsymbol{K},E) | \boldsymbol{k}' m_\beta m_{\beta'} \rangle = \langle \boldsymbol{k} m_\alpha m_{\alpha'} | V | \boldsymbol{k}' m_\beta m_{\beta'} \rangle$$
$$+ \frac{1}{2} \sum_{m_\gamma m_{\gamma'}} \int \frac{d^3q}{(2\pi)^3} \langle \boldsymbol{k} m_\alpha m_{\alpha'} | V | \boldsymbol{q} m_\gamma m_{\gamma'} \rangle \frac{\theta(|\boldsymbol{q}+\boldsymbol{K}/2| - k_F)\ \theta(|\boldsymbol{K}/2-\boldsymbol{q}| - k_F)}{E - \varepsilon(\boldsymbol{q}+\boldsymbol{K}/2) - \varepsilon(\boldsymbol{K}/2-\boldsymbol{q}) + i\eta} \langle \boldsymbol{q} m_\gamma m_{\gamma'} | G(\boldsymbol{K},E) | \boldsymbol{k}' m_\beta m_{\beta'} \rangle$$

- Brueckner G-matrix but Bethe-Goldstone equation...
- Dispersion relation

$$\begin{aligned} \langle \boldsymbol{k} m_\alpha m_{\alpha'} | G(\boldsymbol{K},E) | \boldsymbol{k}' m_\beta m_{\beta'} \rangle &= \langle \boldsymbol{k} m_\alpha m_{\alpha'} | V | \boldsymbol{k}' m_\beta m_{\beta'} \rangle - \frac{1}{\pi} \int_{2\varepsilon_F}^{\infty} dE' \, \frac{\mathrm{Im}\ \langle \boldsymbol{k} m_\alpha m_{\alpha'} | \Delta G(\boldsymbol{K},E') | \boldsymbol{k}' m_\beta m_{\beta'} \rangle}{E - E' + i\eta} \\ &\equiv \langle \boldsymbol{k} m_\alpha m_{\alpha'} | V | \boldsymbol{k}' m_\beta m_{\beta'} \rangle + \langle \boldsymbol{k} m_\alpha m_{\alpha'} | \Delta G_\downarrow(\boldsymbol{K},E) | \boldsymbol{k}' m_\beta m_{\beta'} \rangle \end{aligned}$$

- Include HF term in "BHF" self-energy

$$\Sigma_{BHF}(k;E)) = \int \frac{d^3k'}{(2\pi)^3} \frac{1}{\nu} \sum_{m_\alpha m_{\alpha'}} \theta(k_F - k') \langle \tfrac{1}{2}(\boldsymbol{k}-\boldsymbol{k}') m_\alpha m_{\alpha'} | G(\boldsymbol{k}+\boldsymbol{k}'; E+\varepsilon(\boldsymbol{k}')) | \tfrac{1}{2}(\boldsymbol{k}-\boldsymbol{k}')\ m_\alpha m_{\alpha'} \rangle$$

- Below Fermi energy: no imaginary part

BHF

- DE for k < k_F yields solutions at

$$\varepsilon_{BHF}(k) = \frac{\hbar^2 k^2}{2m} + \Sigma_{BHF}(k; \varepsilon_{BHF}(k))$$

- with strength < 1
- Since there is no imaginary part below the Fermi energy, no momenta above k_F can admix -> problem with particle number
- Only sp energy is determined self-consistently
- Choice of auxiliary potential
  - Standard $U_s(k) = \Sigma_{BHF}(k; \varepsilon_{BHF}(k))$ only for k < k_F (0 above)
  - Continuous $U_c(k) = \Sigma_{BHF}(k; \varepsilon_{BHF}(k))$ all k
- Only one calculation of G-matrix for standard choice
- Iterations for continuous choice

BHF

- Propagator $G^{BHF}(k;E) = \dfrac{\theta(k-k_F)}{E-\varepsilon_{BHF}(k)+i\eta} + \dfrac{\theta(k_F-k)}{E-\varepsilon_{BHF}(k)-i\eta}$
- Energy

$$\frac{E_0^A}{A} = \frac{\nu}{2\rho}\int \frac{d^3k}{(2\pi)^3}\left(\frac{\hbar^2k^2}{2m}+\varepsilon_{BHF}\right)\theta(k-k_F)$$

- Rewrite using on-shell self-energy

$$\frac{E_0^A}{A} = \frac{4}{\rho}\int \frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m} + \frac{1}{2\rho}\int \frac{d^3k}{(2\pi)^3}\int \frac{d^3k'}{(2\pi)^3}\sum_{m_\alpha m_{\alpha'}}\theta(k_F-k)\theta(k_F-k')$$

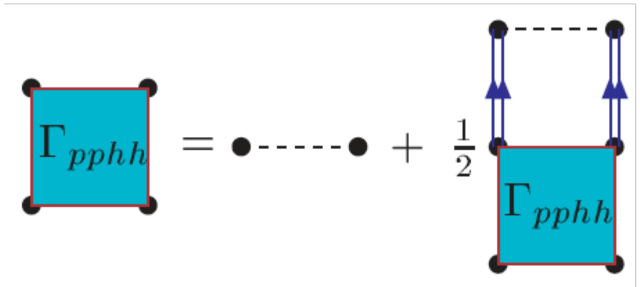
$$\left\langle \tfrac{1}{2}(\boldsymbol{k}-\boldsymbol{k}')\; m_\alpha m_{\alpha'}\right| G(\boldsymbol{k}+\boldsymbol{k}';\varepsilon_{BHF}(k)+\varepsilon_{BHF}(k'))\left|\tfrac{1}{2}(\boldsymbol{k}-\boldsymbol{k}')\; m_\alpha m_{\alpha'}\right\rangle$$

- First term: kinetic energy free Fermi gas
- Compare

$$\begin{aligned} E^{HF} &= \frac{1}{2}\sum_{\boldsymbol{p}}\theta(p_F-p)\left[\frac{\boldsymbol{p}^2}{2m}+\varepsilon^{HF}(p)\right] \\ &= T_{FG} + \frac{1}{2}\sum_{\boldsymbol{pp}'}\theta(p_F-p)\theta(p_F-p')\left\langle \boldsymbol{pp}'\right| V \left|\boldsymbol{pp}'\right\rangle \end{aligned}$$

- so BHF obtained by replacing V by G

Lowest-order Brueckner theory (two hole lines)



G^f_{BG} angle-average of

$$G^f_{BG}(k_1,k_2;E) = \frac{\theta(k_1 - k_F)\theta(k_2 - k_F)}{E - \varepsilon(k_1) - \varepsilon(k_2) + i\eta}$$

$$\langle k\ell|G^{JST}(K,E)|k'\ell'\rangle = \langle k\ell|V^{JST}|k'\ell'\rangle + \frac{1}{2}\sum_{\ell''}\int_0^\infty \frac{dq}{(2\pi)^3} q^2 \langle k\ell|V^{JST}|q\ell''\rangle G^f_{BG}(q;K,E)\langle q\ell''|G^{JST}(K,E)|k'\ell'\rangle$$

Spectrum
$$\varepsilon_{BHF}(k) = \frac{\hbar^2k^2}{2m} + \Sigma_{BHF}(k;\varepsilon_{BHF}(k))$$

$k < k_F \Rightarrow$ standard choice

all $k \Rightarrow$ continuous choice

Self-energy
$$\Sigma_{BHF}(k;E) = \frac{1}{\nu}\sum_{m,m'}\int \frac{d^3k'}{(2\pi)^3}\theta(k_F - k')\langle \vec{k}\vec{k}'mm'|G(\vec{k}+\vec{k}';E+\varepsilon_{BHF}(k'))|\vec{k}\vec{k}'mm'\rangle$$

Energy
$$\frac{E}{A} = \frac{4}{\rho}\int \frac{d^3k}{(2\pi)^3}\theta(k_F - k)\frac{\hbar^2k^2}{2m}$$
$$+\frac{1}{2\rho}\sum_{m,m'}\int \frac{d^3k}{(2\pi)^3}\theta(k_F - k)\int \frac{d^3k'}{(2\pi)^3}\theta(k_F - k')\langle \vec{k}\vec{k}'mm'|G(\vec{k}+\vec{k}';\varepsilon_{BHF}(k)+\varepsilon_{BHF}(k'))|\vec{k}\vec{k}'mm'\rangle$$

Old pain and suffering!

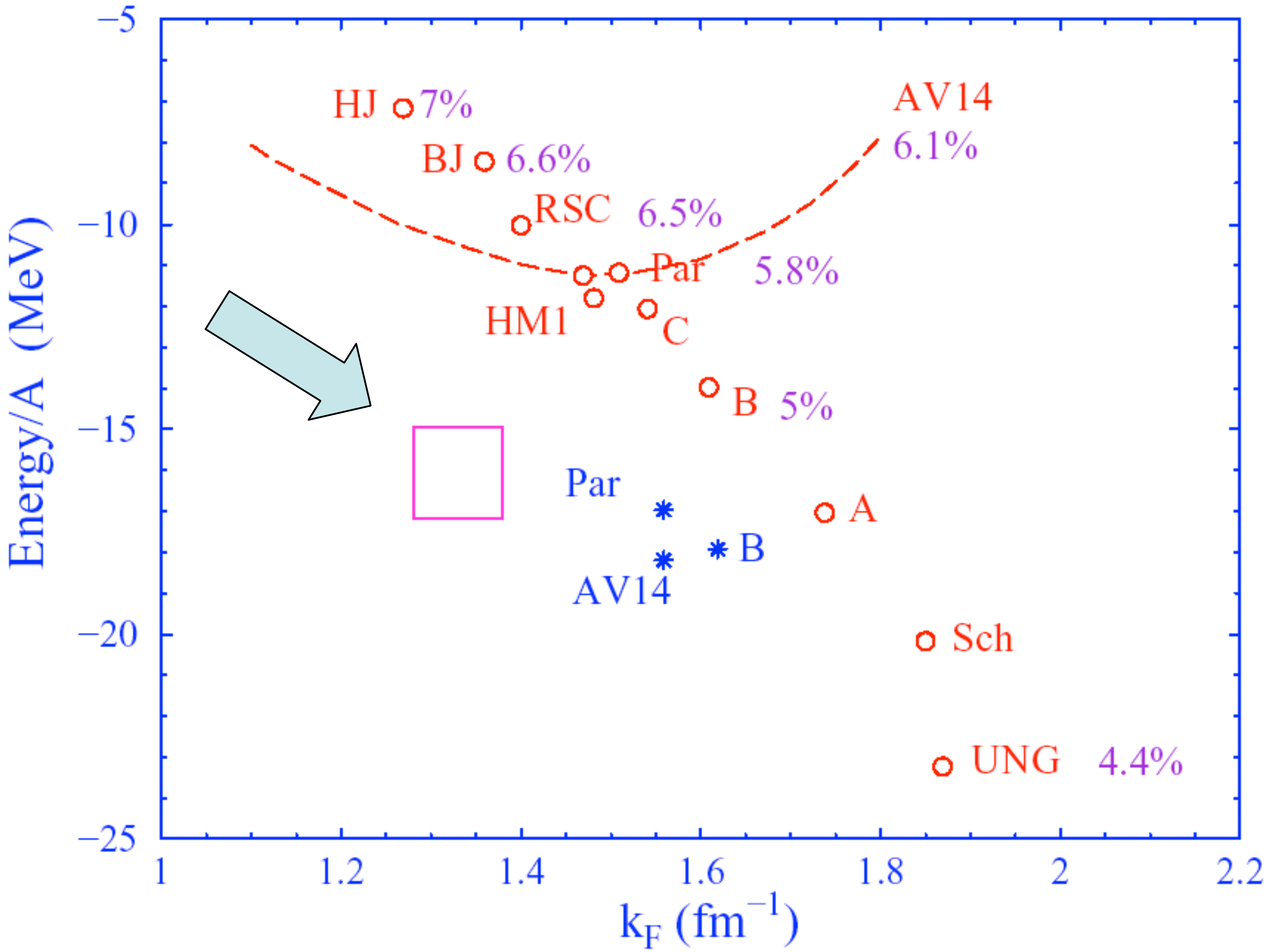
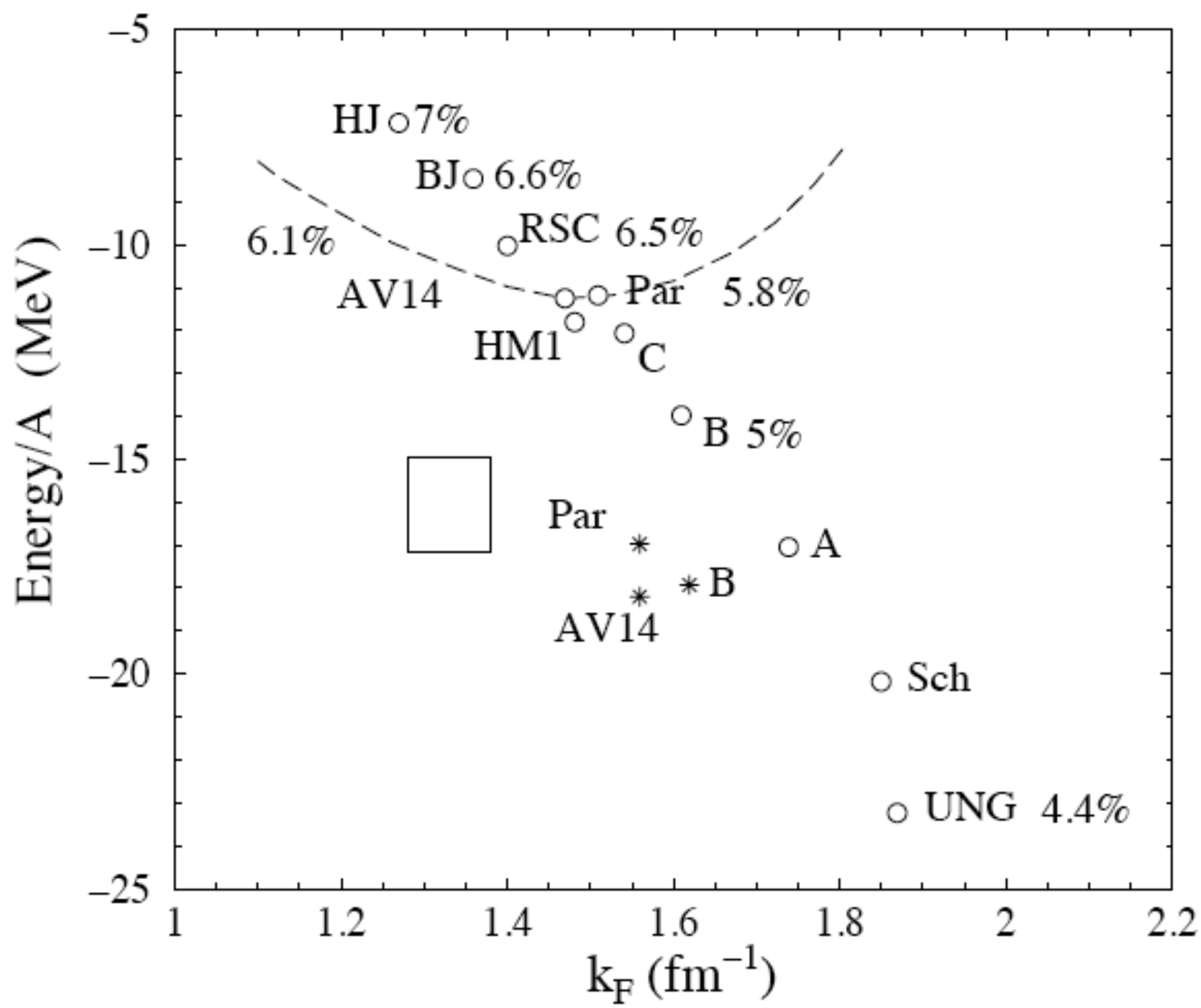


Figure adapted from Marcello Baldo (Catania)

BHF

- Binding energy usually within 10 MeV from empirical volume term in the mass formula even for very strong repulsive cores
- Repulsion always completely cancelled by higher-order terms
- Minimum in density never coincides with empirical value when binding OK -> Coester band



Location of minimum determined by deuteron D-state probability

Some remarks

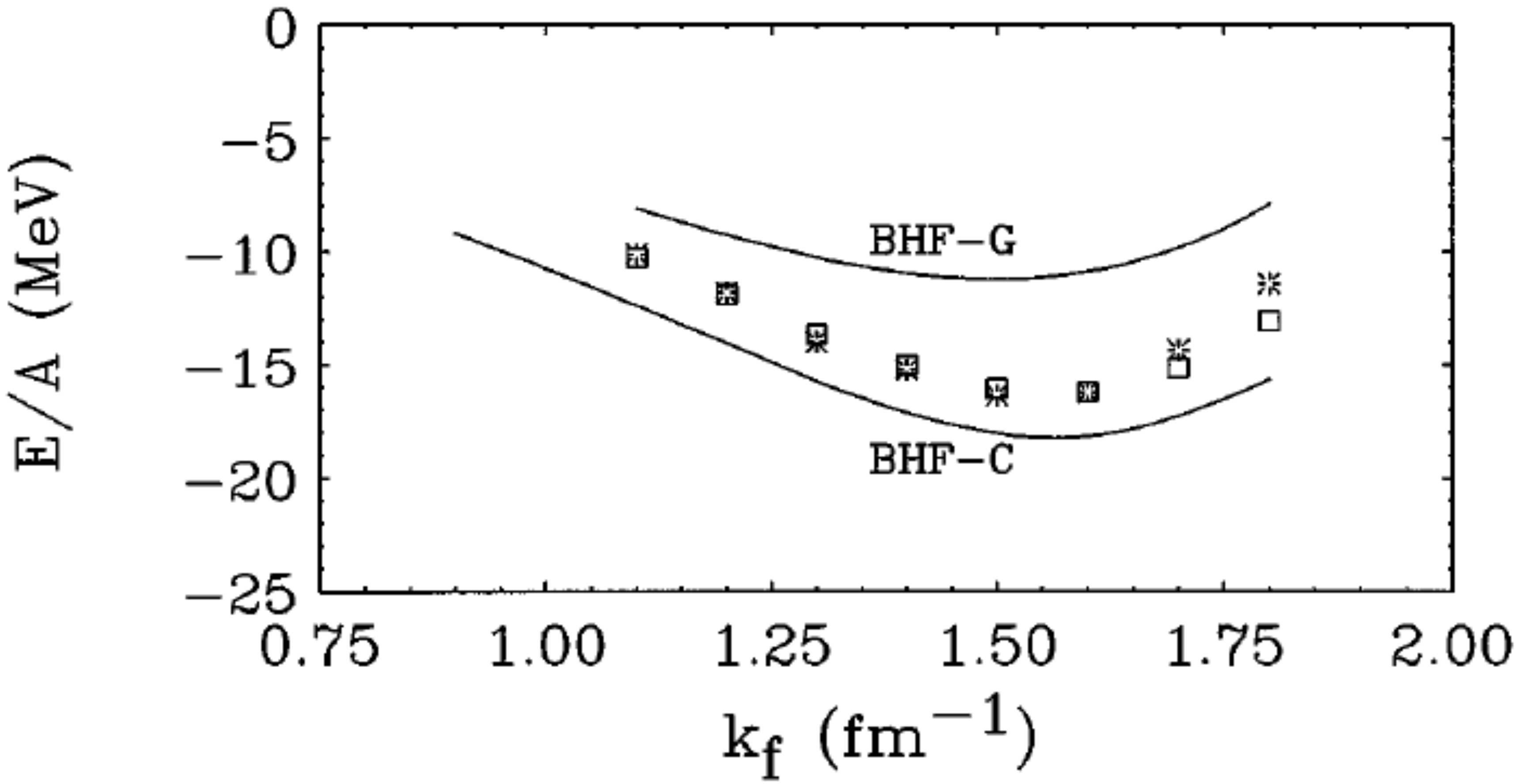
- Variational results (end 1970s) gave more binding than G-matrix calculations
- Interest in convergence of Brueckner approach
- Bethe et al.: hole-line expansion had already been developed
- G-matrix: sums all energy terms with 2 independent hole lines (noninteracting ...)
- Dominant for low-density
- Phase space arguments suggests to group all terms with 3 independent hole lines as the next contribution
- Requires technique from 3-body problem first solved by Faddeev -> Bethe-Faddeev summation
- First implemented by Ben Day
- Including these terms generates minima indicated by ✴ in figure (Baldo et al.)
- Better but not yet good enough

More

- Variational results and 3-hole-line results more or less in agreement
- Baldo et al. also calculated 3-hole-line terms with continuous choice for auxiliary potential and found that results do not depend on choice of auxiliary potential, furthermore 2-hole-line with continuous choice is already "almost" sufficient!
- Conclusion: convergence appears OK for a given realistic two-body interaction for the energy per particle
- Other quantities —> not always consistent (Hugenholtz-Van Hove)
- Still nuclear matter saturation problem!

Results hole-line expansion

- Original papers B.D.Day, PRC 24, 1203 (1981) & PRL47, 226 (1981)
- Important confirmation Baldo et al. PRL81, 1584 (1998)

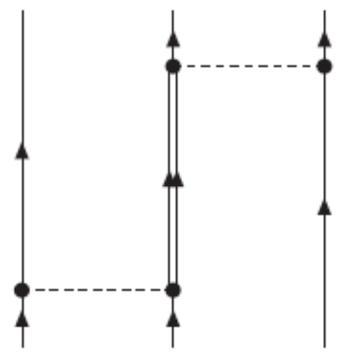


Conclusion

- Given a realistic NN interaction, the energy of the ground state of nuclear matter can be calculated in a systematic way
- Results at moderate densities converge to the same result for different choices for the auxiliary potential
- Continuous choice at the BHF level already a good approximation
- Different realistic interactions yield a saturation density that is too high and the amount of binding is reasonable or somewhat too large

- Now what?

Possible solutions

- Include three-body interactions: inevitable on account of isobar
  - Simplest diagram:  space of nucleons -> 3-body force
  - Inclusion in nuclear matter still requires phenomenology to get saturation right
  - Also needed for few-body nuclei; there is some incompatibility
- Include aspects of relativity
  - Dirac-BHF approach: ad hoc adaptation of BHF to nucleon spinors
  - Physical effect: coupling to scalar-isoscalar meson reduced with density
  - Antiparticles? Dirac sea? Three-body correlations?
  - Spin-orbit splitting in nuclei OK
  - Nucleons less correlated with higher density? (compare liquid ^{3}He)

Finite nuclei

- What can we learn from finite nuclei

- Almost exact calculations possible for light nuclei
- Not restricted to NN interactions
- Can include NNN interactions

- But interactions must be local for Monte Carlo results!

From a talk of Bob Wiringa (Argonne National Lab)

Variational Monte Carlo

Minimize expectation value of H

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Trial function (s-shell nuclei)

$$|\Psi_V\rangle = \left[1 + \sum_{i<j<k} U_{ijk}^{TNI}\right] \left[\mathcal{S} \prod_{i<j} (1 + U_{ij})\right] |\Psi_J\rangle$$

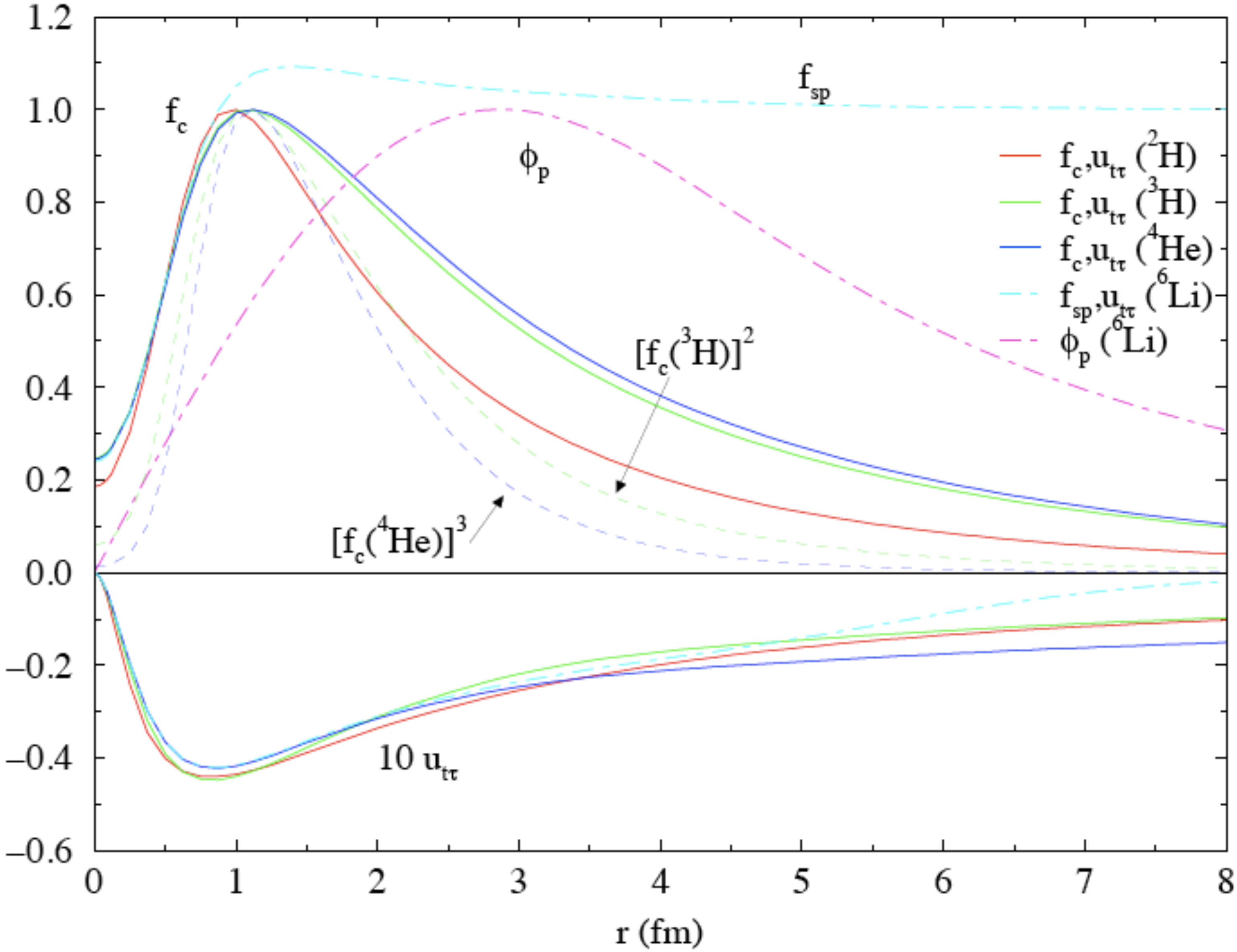
$$|\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij})\right] |\Phi_A(JMTT_3)\rangle$$

$$|\Phi_d(1100)\rangle = \mathcal{A}|\uparrow p \uparrow n\rangle \;\; ; \;\; |\Phi_\alpha(0000)\rangle = \mathcal{A}|\uparrow p \downarrow p \uparrow n \downarrow n\rangle$$

$$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O_{ij}^p \;\; ; \;\; U_{ijk}^{TNI} = -\epsilon V_{ijk}(\tilde{r}_{ij}, \tilde{r}_{jk}, \tilde{r}_{ki})$$

Functions $f_c(r_{ij})$ and $u_p(r_{ij})$ obtained from coupled differential equations with v_{ij} .

Correlation functions



Green's Function Monte Carlo

Projects out lowest energy state from variational trial function

$$\begin{aligned}\Psi(\tau) = \exp[-(H-E_0)\tau]\Psi_V &= \sum_n \exp[-(E_n-E_0)\tau]a_n\psi_n \\ \Psi(\tau\to\infty) &= a_0\psi_0\end{aligned}$$

Evaluation of $\Psi(\tau)$ done stochastically in small time steps $\Delta\tau$

$$\Psi(\mathbf{R}_n,\tau) = \int G(\mathbf{R}_n,\mathbf{R}_{n-1})\cdots G(\mathbf{R}_1,\mathbf{R}_0)\Psi_V(\mathbf{R}_0)d\mathbf{R}_{n-1}\cdots d\mathbf{R}_0$$

using the short-time propagator accurate to order $(\Delta\tau)^3$ (V_{ijk} term omitted for simplicity)

$$G_{\alpha\beta}(\mathbf{R},\mathbf{R}') = e^{E_0\Delta\tau}G_0(\mathbf{R},\mathbf{R}')\langle\alpha|\left[\mathcal{S}\prod_{i<j}\frac{g_{ij}(\mathbf{r}_{ij},\mathbf{r}'_{ij})}{g_{0,ij}(\mathbf{r}_{ij},\mathbf{r}'_{ij})}\right]|\beta\rangle$$

where the free many-body propagator is

$$G_0(\mathbf{R},\mathbf{R}') = \langle\mathbf{R}|e^{-K\Delta\tau}|\mathbf{R}'\rangle = \left[\sqrt{\frac{m}{2\pi\hbar^2\Delta\tau}}\right]^{3A}\exp\left[\frac{-(\mathbf{R}-\mathbf{R}')^2}{2\hbar^2\Delta\tau/m}\right]$$

and $g_{0,ij}$ and g_{ij} are the free and exact two-body propagators

$$g_{ij}(\mathbf{r}_{ij},\mathbf{r}'_{ij}) = \langle\mathbf{r}_{ij}|e^{-H_{ij}\Delta\tau}|\mathbf{r}'_{ij}\rangle$$

Mixed estimates

$$\langle O(\tau)\rangle = \frac{\langle\Psi(\tau)|O|\Psi(\tau)\rangle}{\langle\Psi(\tau)|\Psi(\tau)\rangle} \approx \langle O(\tau)\rangle_{\rm Mixed} + [\langle O(\tau)\rangle_{\rm Mixed} - \langle O\rangle_V]$$

$$\langle O(\tau)\rangle_{\rm Mixed} = \frac{\langle\Psi_V|O|\Psi(\tau)\rangle}{\langle\Psi_V|\Psi(\tau)\rangle} \quad ; \quad \langle H(\tau)\rangle_{\rm Mixed} = \frac{\langle\Psi(\tau/2)|H|\Psi(\tau/2)\rangle}{\langle\Psi(\tau/2)|\Psi(\tau/2)\rangle} \geq E_0$$

Propagator cannot contain p^2, L^2, or $(\mathbf{L}\cdot\mathbf{S})^2$ operators:

$G_{\beta\alpha}(\mathbf{R}',\mathbf{R})$ has only v_8'

$\langle v_{18} - v_8'\rangle$ computed perturbatively with extrapolation (small for AV18)

Fermion sign problem limits maximum τ:

$G_{\beta\alpha}(\mathbf{R}',\mathbf{R})$ brings in lower-energy boson solution

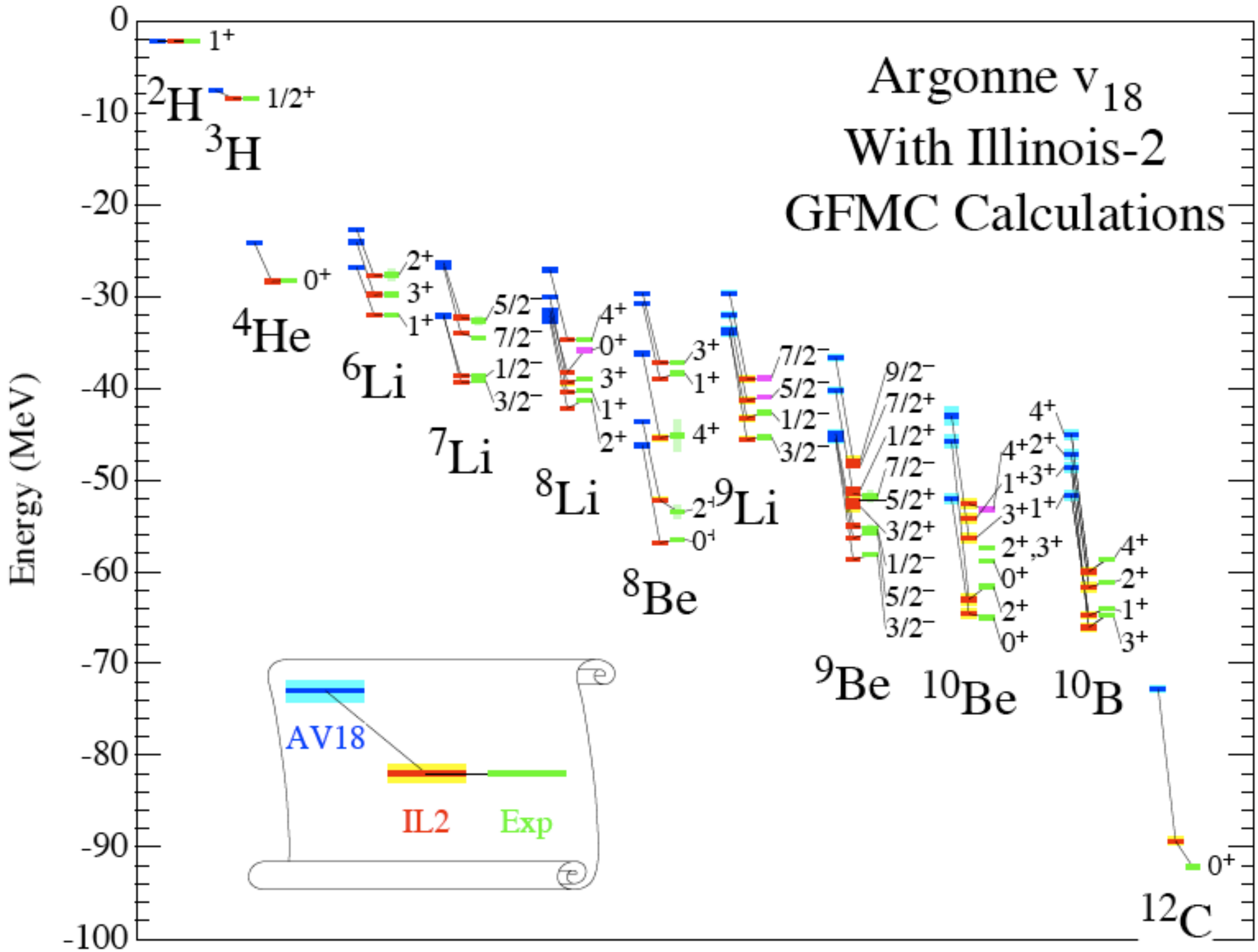
$\langle\Psi_V|H|\Psi(\tau)\rangle$ projects back fermion solution. but statistical errors grow exponentially

Constrained-path propagation, removes steps that have

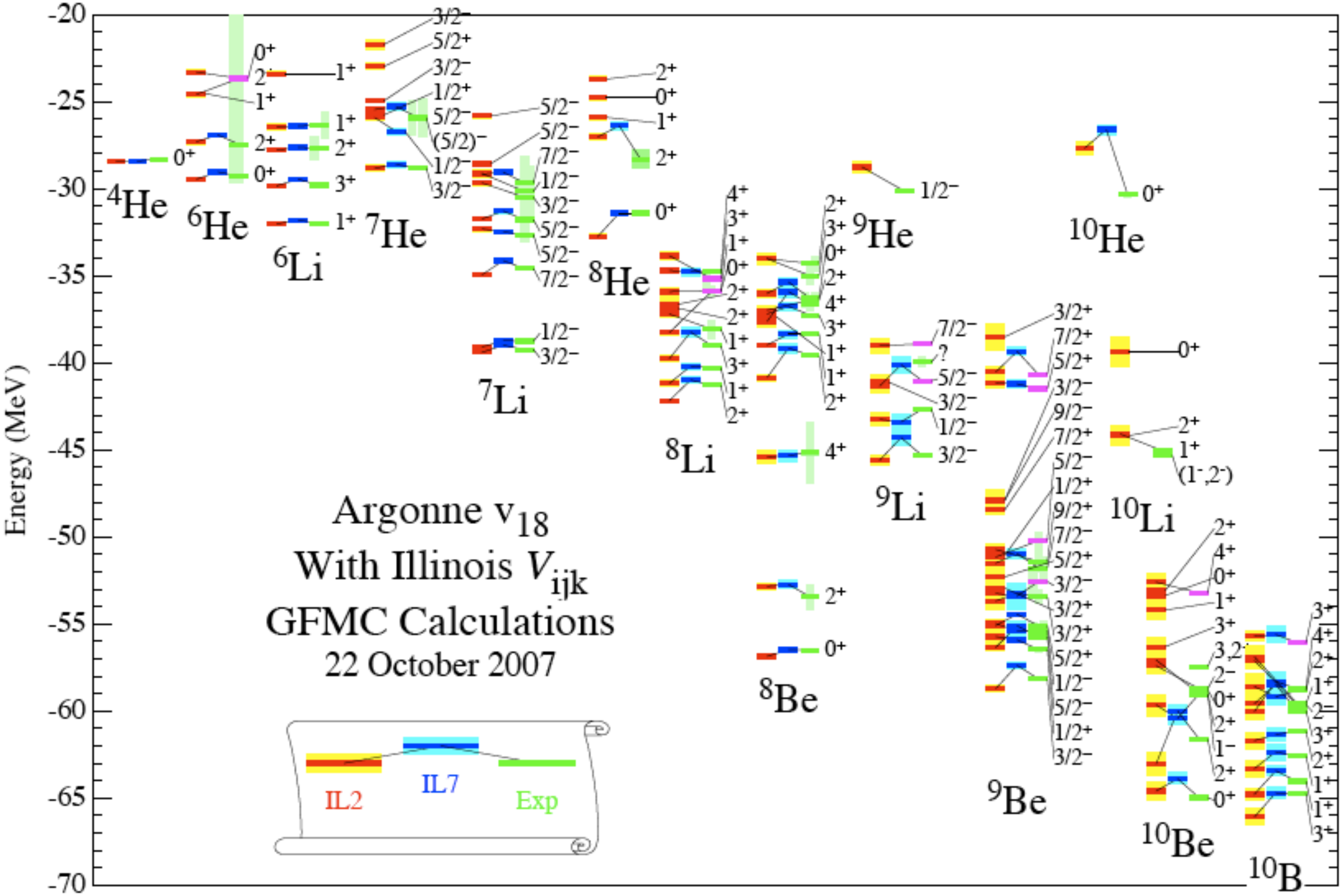
$$\overline{\Psi^\dagger(\tau,\mathbf{R})\Psi(\mathbf{R})} = 0$$

Possible systematic errors reduced by $10 - 20$ unconstrained steps before evaluating observables.

Effect of 3N attractive



More recent tuning 3N



Energy of the ground state & NNN

- Energy sum rule (Migdal, Galitski & Koltun)

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

- **Not** part of fit because it can only make a statement about the two-body contribution
- Result:
  - **DOM** ---> 7.91 MeV/A T/A ---> 22.64 MeV/A
  - 10% of particles (momenta > 1.4 fm-1) provide ~⅔ of the binding energy!
  - Exp. 8.55 MeV/A
  - Three-body ---> 0.64 MeV/A "attraction" —> 1.28 MeV/A "repulsion"
  - Argonne GFMC ~ 1.5 MeV/A attraction for three-body <--> Av18

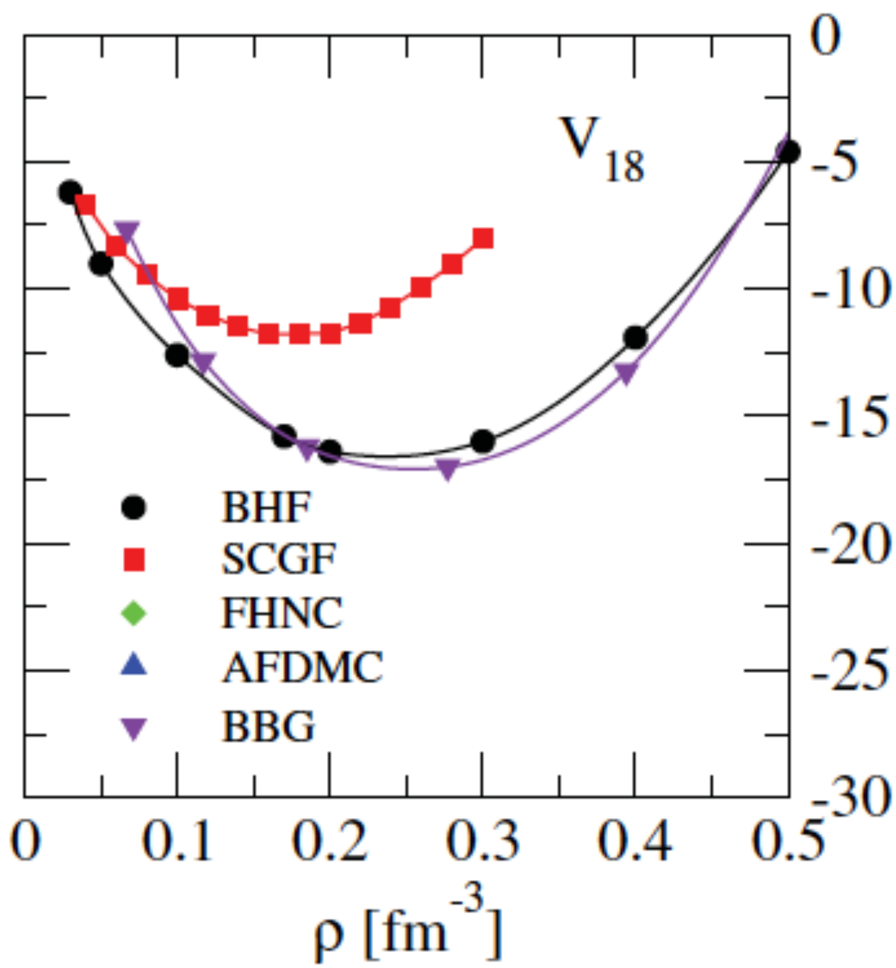
$$\begin{aligned} E_0^N &= \langle \Psi_0^N | \hat{H} | \Psi_0^N \rangle \\ &= \frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F^-} dE \sum_{\alpha,\beta}\{\langle \alpha | T | \beta \rangle + E\ \delta_{\alpha,\beta}\}\ \mathrm{Im}\ G(\beta,\alpha;E) - \frac{1}{2}\langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle \end{aligned}$$

with three-body interaction

reactions and structure

But how does this square with nuclear matter?

- From PRC 86, 064001 (2012)



- Requires a repulsive NNN at high density
- But: Argonne group <—> nuclear matter?

Physics of saturation

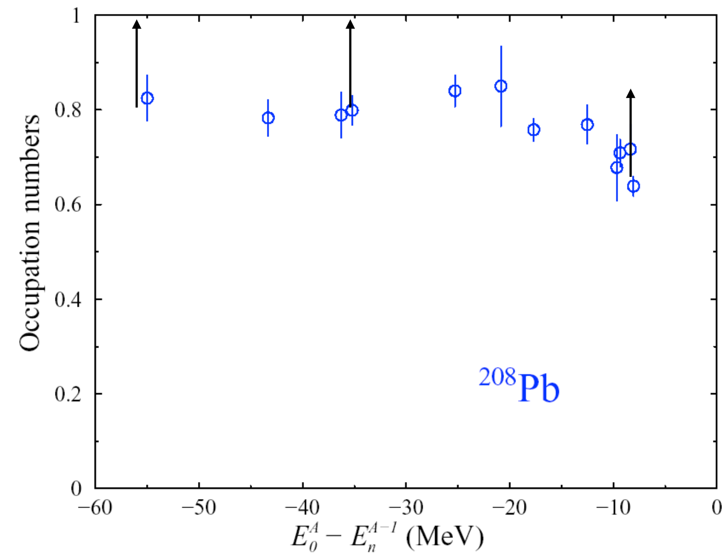
- How do we determine the saturation density
  - SRC
  - LRC
  - what are LRC in nuclei and nuclear matter

- How do we extract the binding energy at saturation

Saturation density and SRC

- Saturation density related to nuclear charge density at the origin. Data for ^{208}Pb $\Rightarrow A/Z\, {}^{*}\rho_{ch}(0) = 0.16\ fm^{-3}$
- Charge at the origin determined by protons in s states
- Occupation of 0s and 1s totally dominated by SRC as can be concluded from recent analysis of $^{208}Pb(e,e'p)$ data and theoretical calculations of occupation numbers in nuclei and nuclear matter.
- Depletion of 2s proton also dominated by SRC:

  15% of the total depletion of 25% ($n_{2s} = 0.75$)



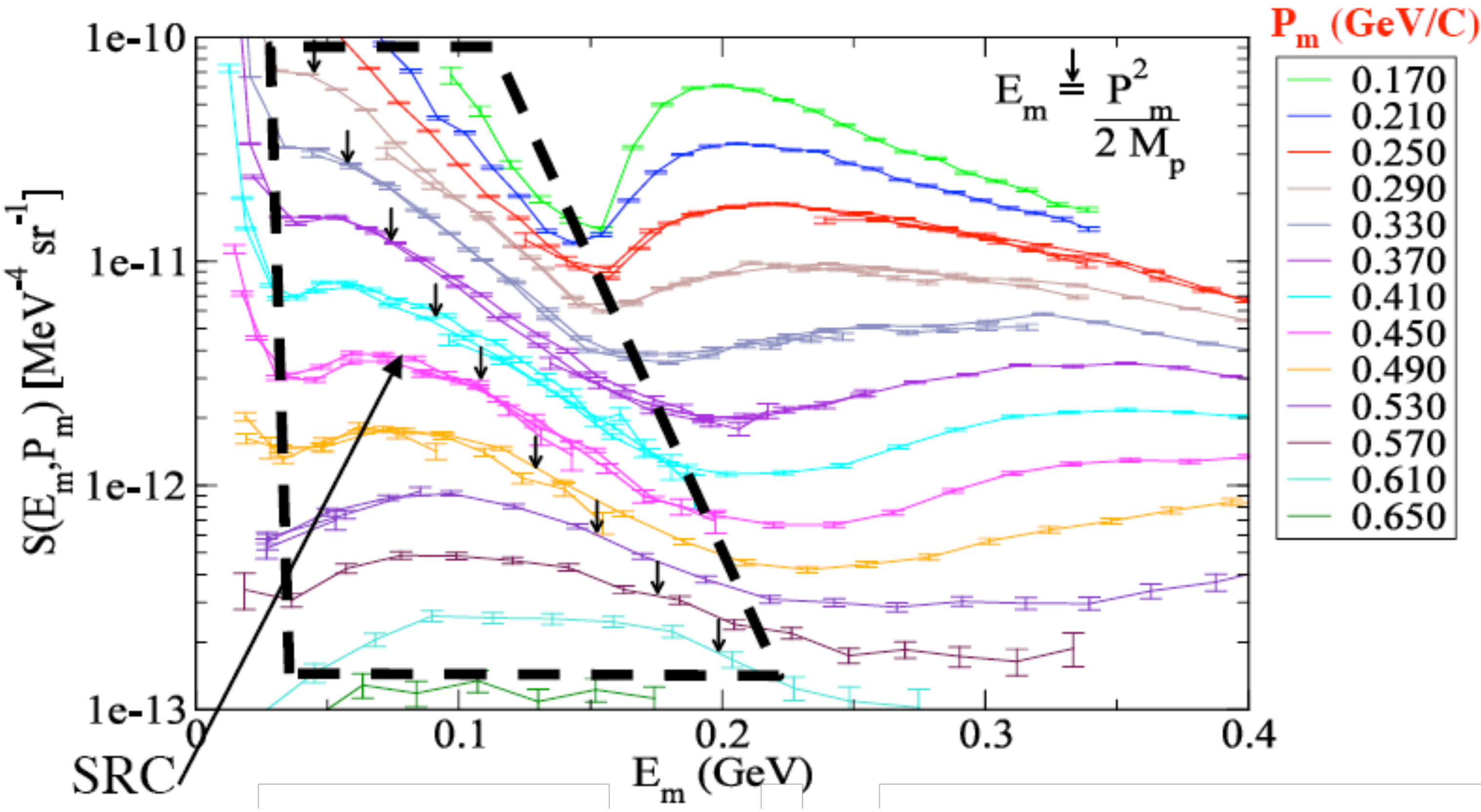
- Conclusion: **Nuclear saturation dominated by SRC**

  and therefore high-momentum components

Green's function V

What are the rest of the protons doing?

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



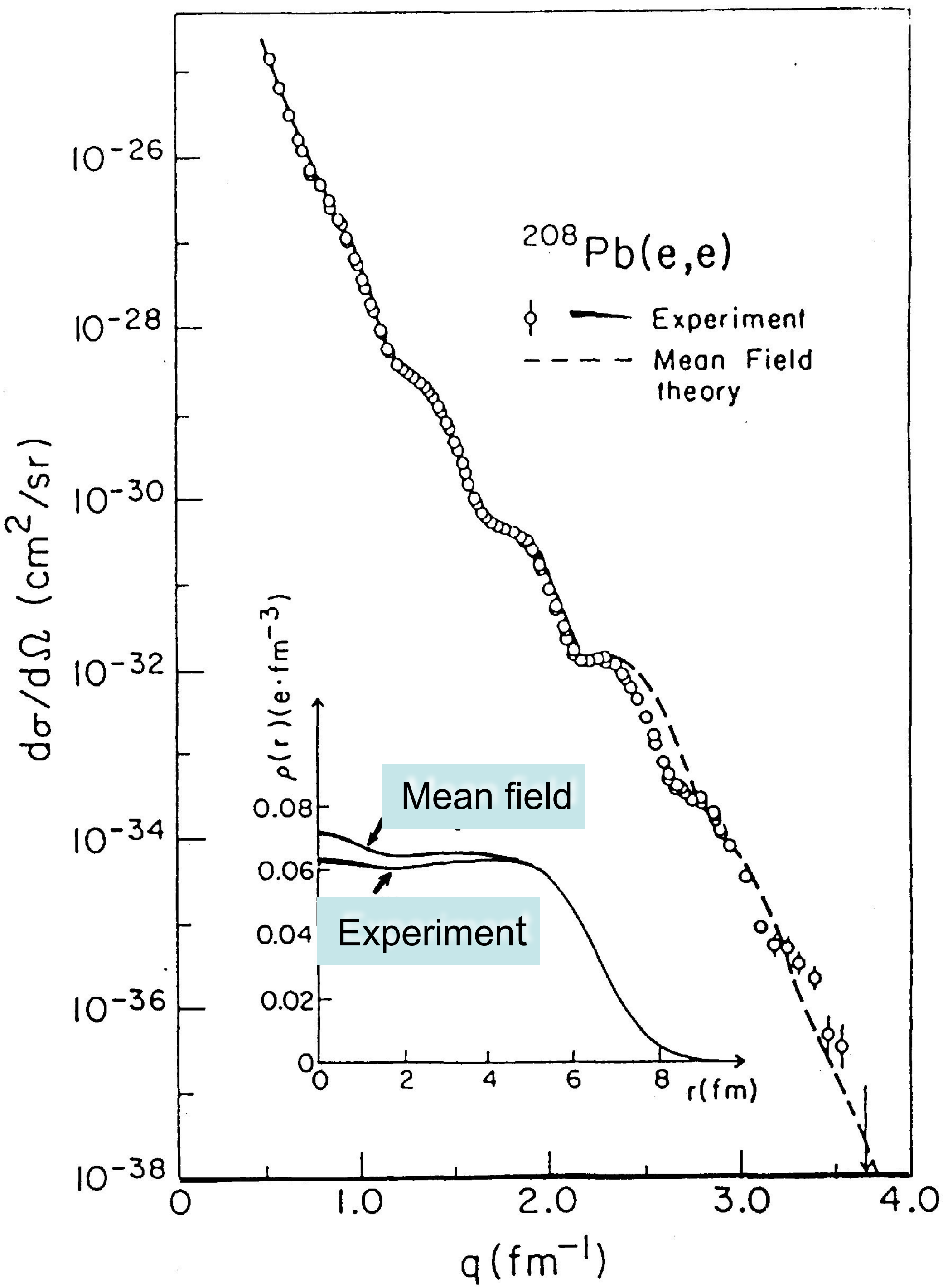
^{12}C

- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

$\Rightarrow$ 0.6 protons for ^{12}C 10% —> important contribution to binding!

^{16}O PRC51,3040(1995) Green's function V

Elastic electron scattering from ^{208}Pb

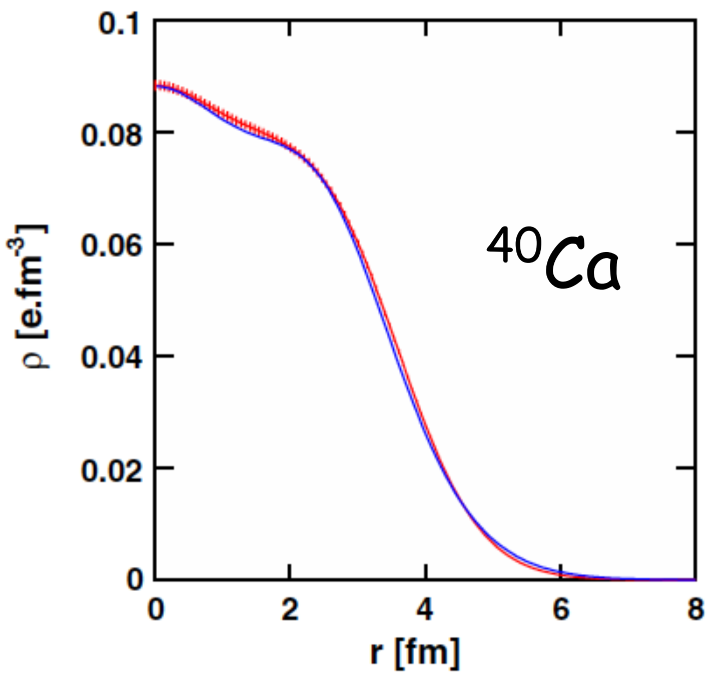


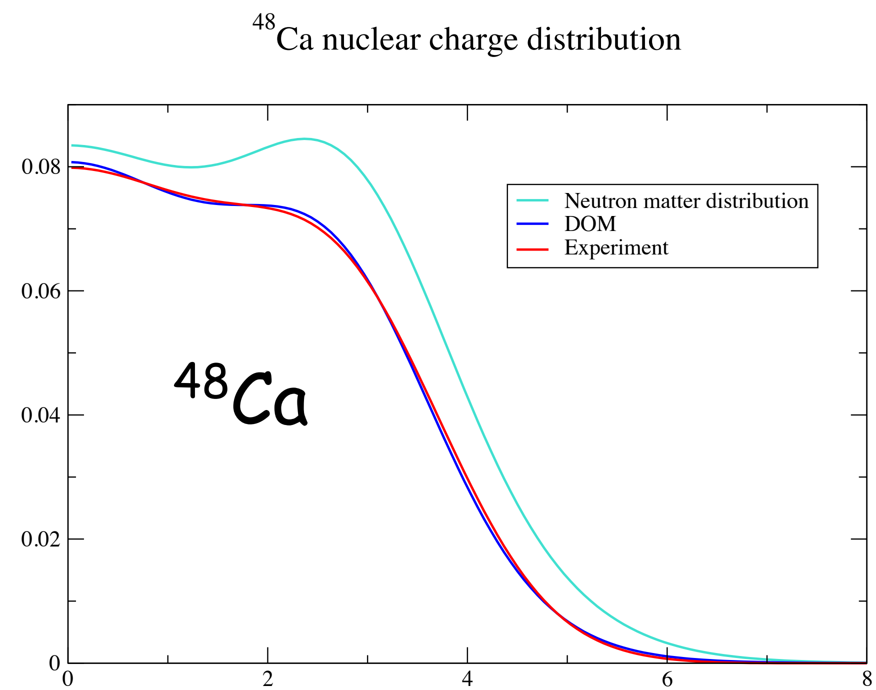
B. Frois et al.

Phys. Rev. Lett. **38**, 152 (1977)

Saturation density <—> Charge density

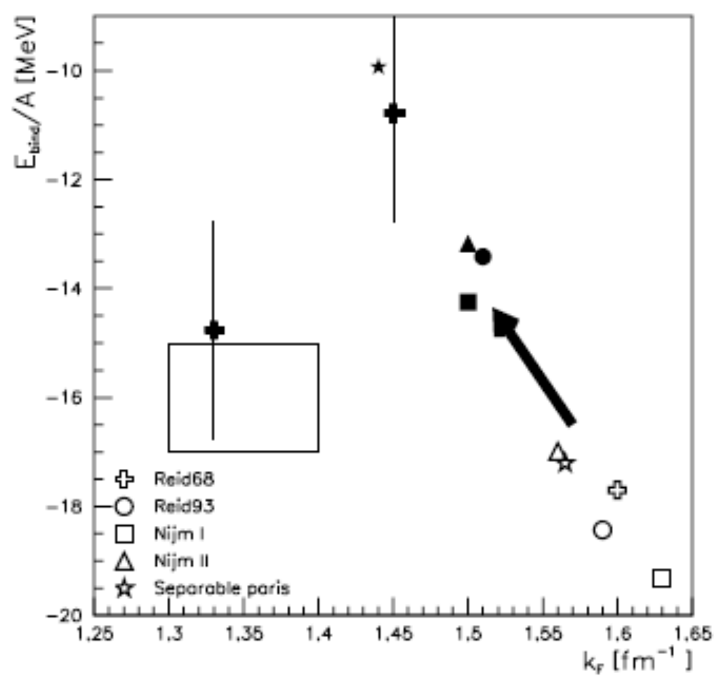
- Experimental results & empirical reproduction by DOM
- A/Z *charge density —> depends on properties of symmetry energy





Personal perspective 2003

- Based on results from (e,e'p) reactions as discussed here
  - nucleons are dressed (substantially) and this should be included in the description of nuclear matter (depletion, high-momentum components in the ground state, propagation w.r.t. correlated ground state <--> BHF?)
  - SRC dominate actual value of saturation density
    - from ^{208}Pb charge density: 0.16 nucleons/fm^3
    - determined from s-shell proton occupancy at small radius
    - occupancy determined mostly by SRC
  - Result for SCGF of ladders
    - Ghent discrete approach
    - St. Louis gaussians
    - ccBHF --> SCGF closer to box
    - do not include LRC!!



Phys. Rev. Lett. 90, 152501 (2003)

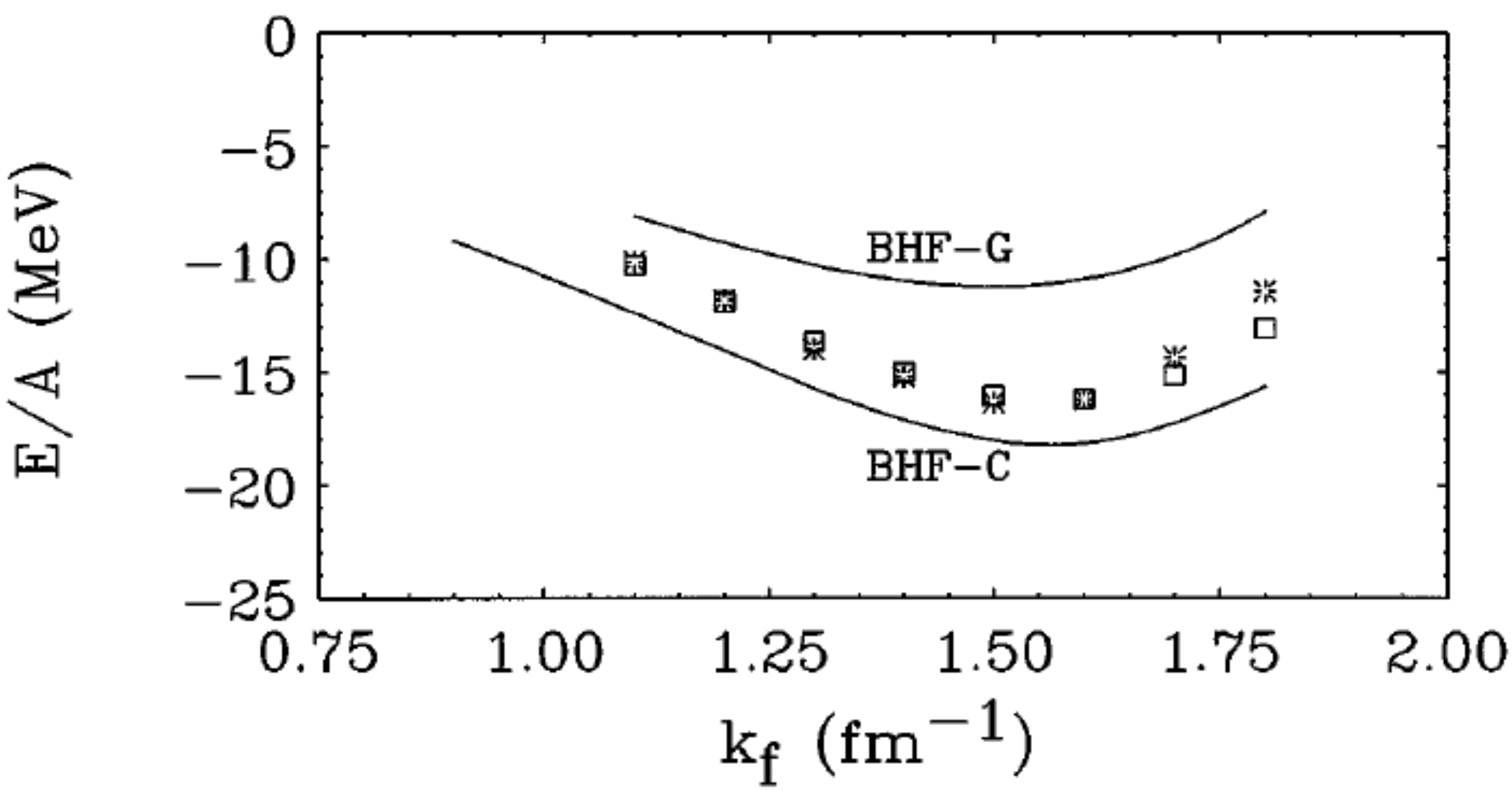
Why can't we get it right?

- Look at hole-line expansion

- Identify LRC contribution to the energy

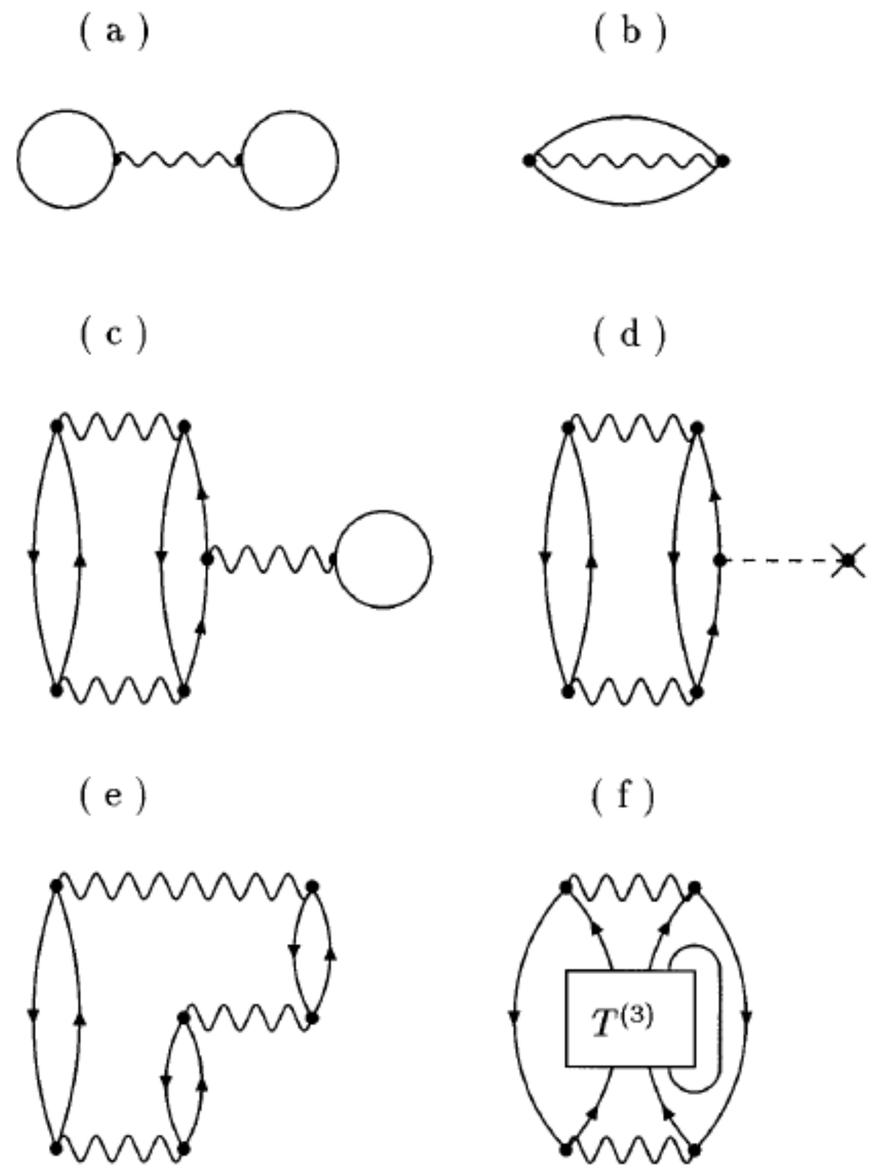
Results hole-line expansion

- Original papers B.D.Day, PRC 24, 1203 (1981) & PRL47, 226 (1981)
- Important confirmation Baldo et al. PRL81, 1584 (1998)

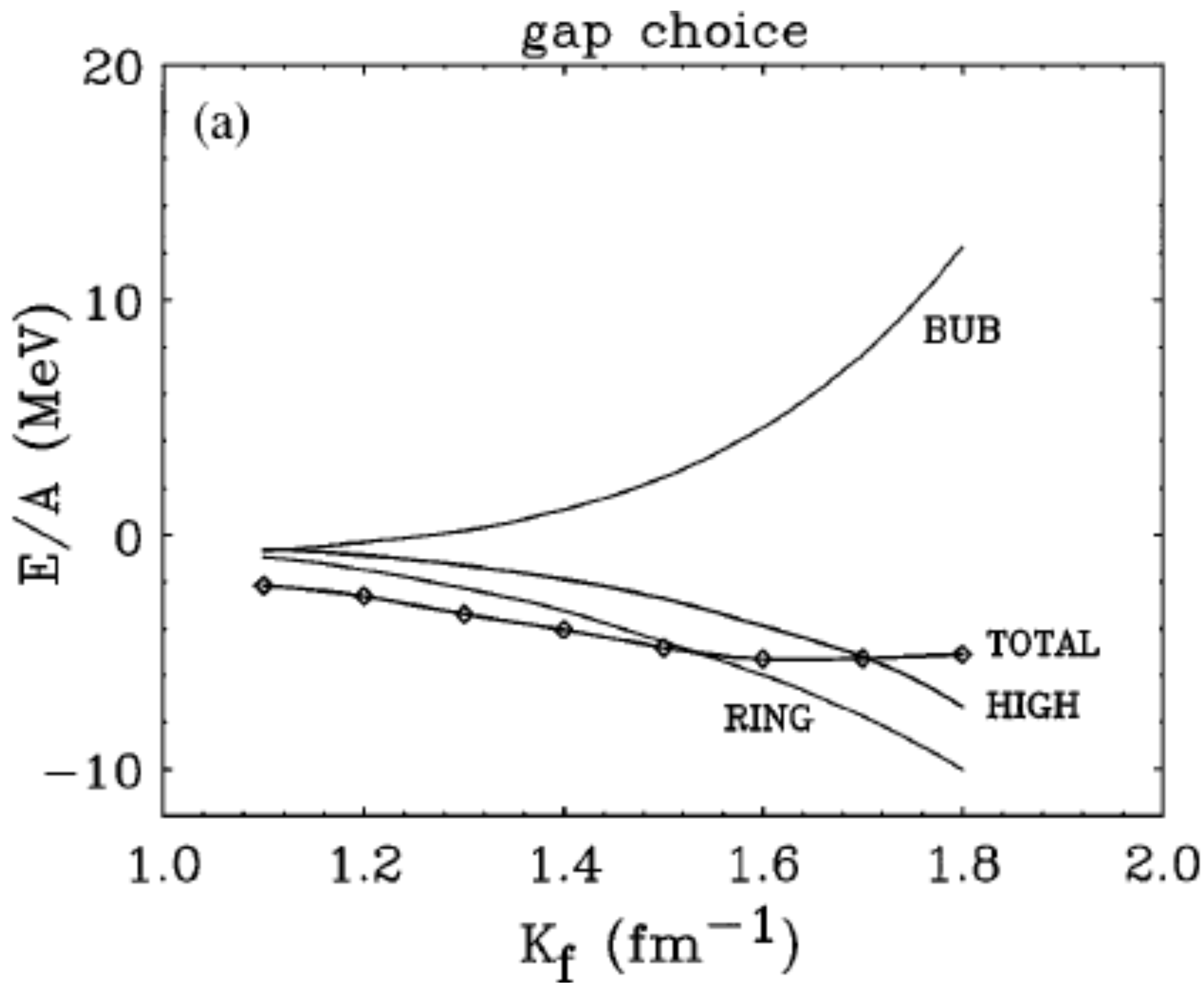


Some ingredients

- Wiggle: G-matrix
- a) + b) = 2 hole-line = BHF
- c) + d) +e) +f) = 3 hole-line
- c) bubble
- d) U insertion for C choice
- e) ring
- f) summed in Bethe-Faddeev



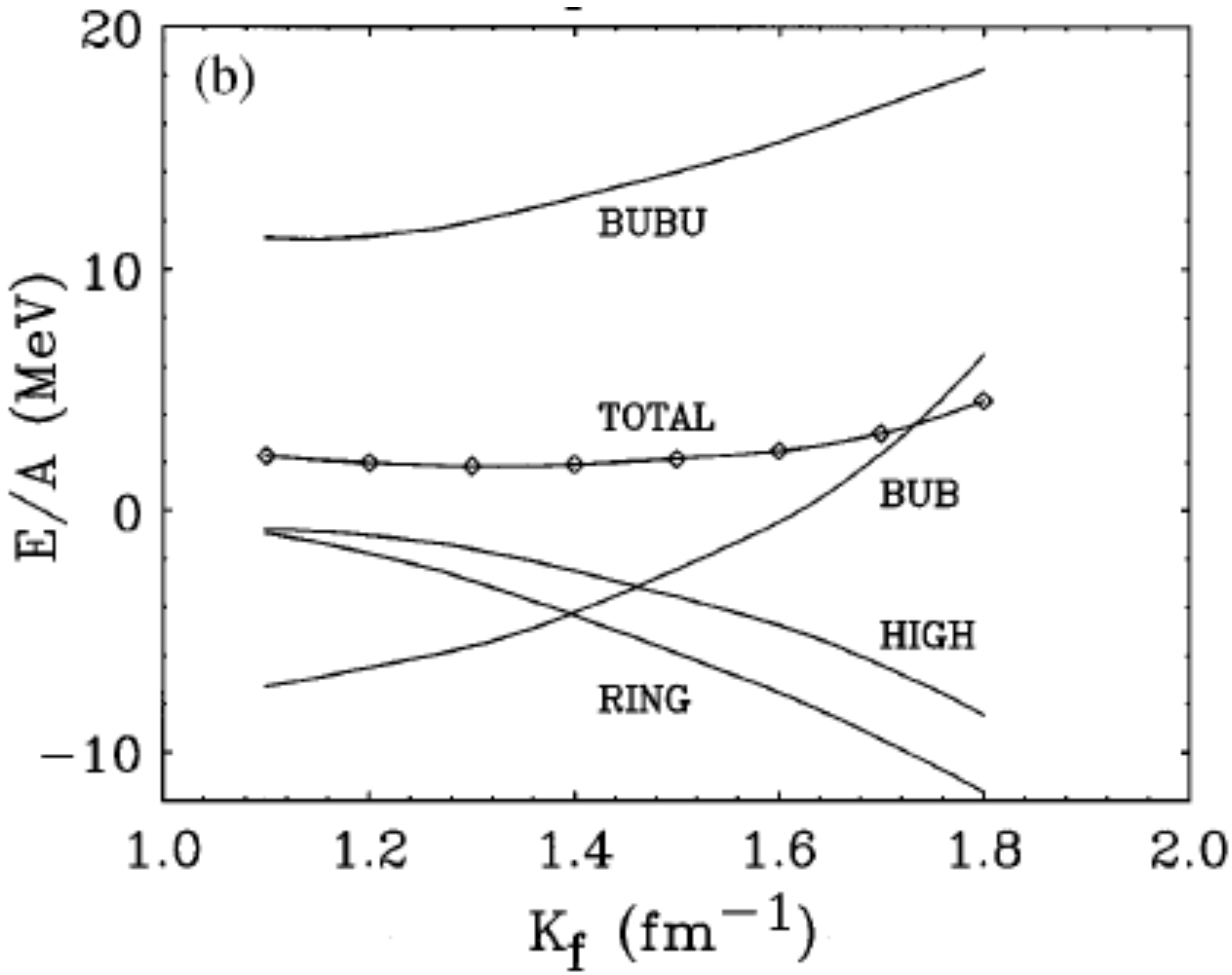
Individual contributions gap choice



- PRL 81, 1584 (1998) Baldo et al.

Continuous choice

- PRL 81, 1584 (1998) Baldo et al.



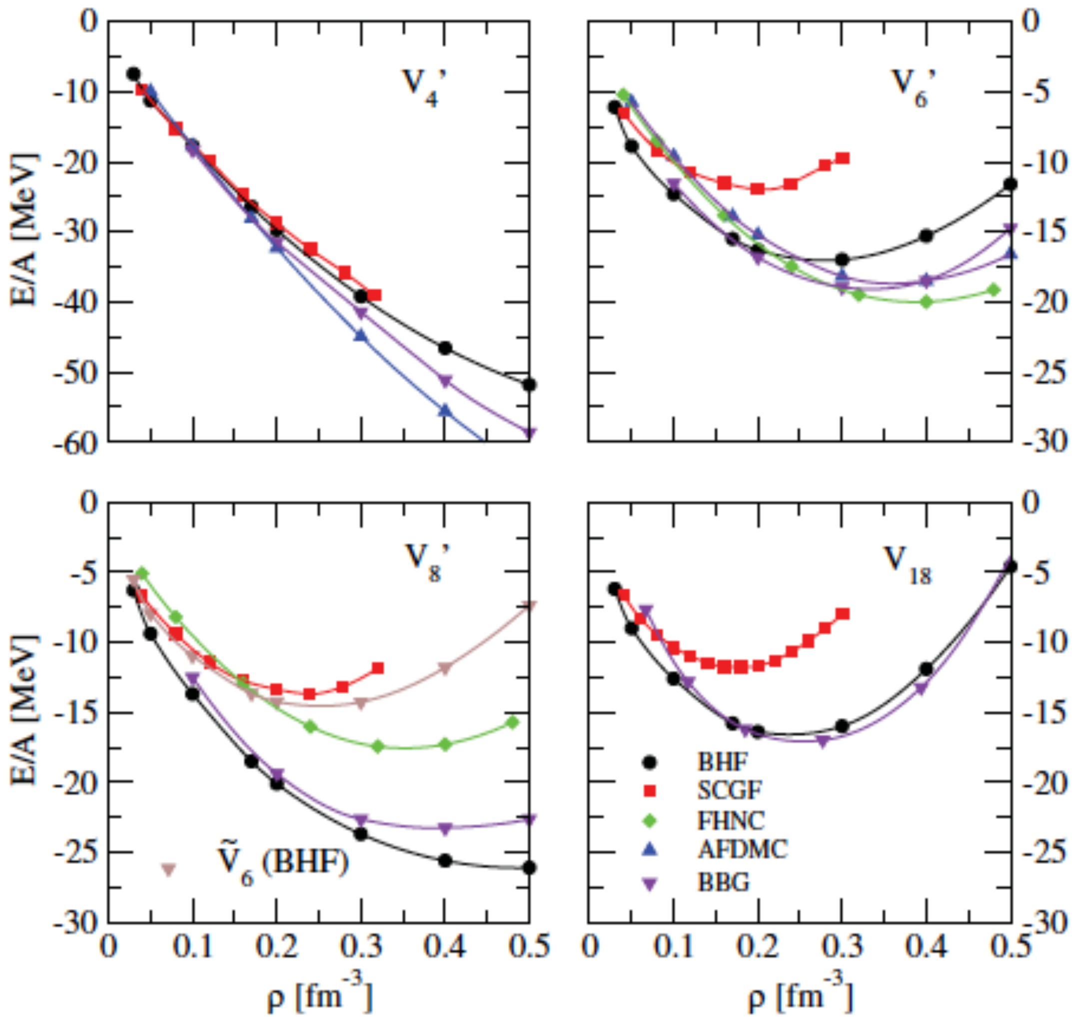
Some comparisons

- SNM

PHYSICAL REVIEW C 86, 064001 (2012)

Comparative study of neutron and nuclear matter with simplified Argonne nucleon-nucleon potentials

M. Baldo,[1] A. Polls,[2] A. Rios,[3] H.-J. Schulze,[1] and I. Vidaña[4]



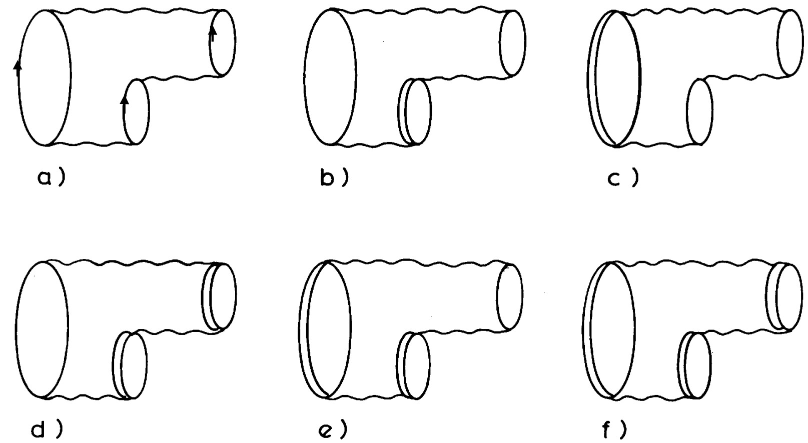
- SCGF Contribution of long-range correlations excluded

What about long-range correlations in nuclear matter?

- Collective excitations in nuclei very different from those in nuclear matter
- Long-range correlations **normally** associated with small q
- Contribution to the energy like $dq\, q^2 \Rightarrow$ very small (except for e-gas)
- Contributions of collective excitations to the binding energy of nuclear matter dominated by pion-exchange induced excitations?!?

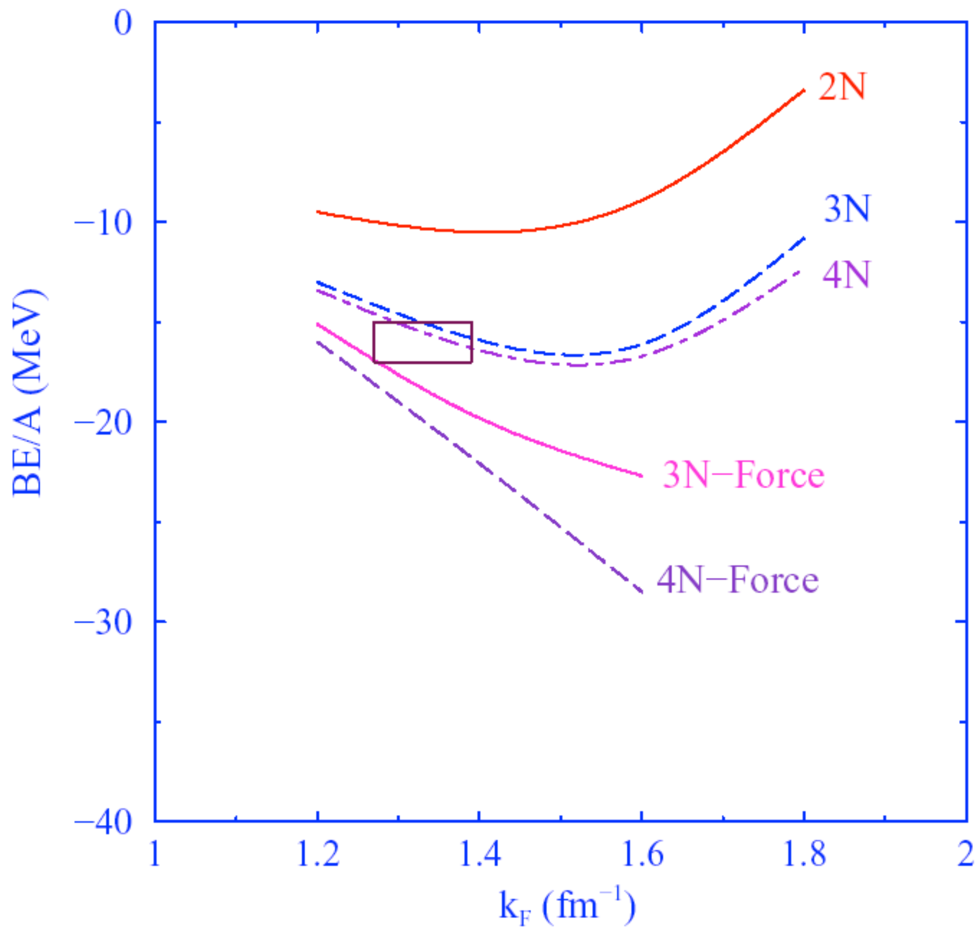
Inclusion of Δ-isobars as "3N-" and "4N-force"

Nucl. Phys. A389, 492 (1982)



k_F [fm^{-1}]	1.0	1.2	1.4	1.6
third order				
a)	-0.303	-1.269	-3.019	-5.384
b)	-0.654	-1.506	-2.932	-5.038
c)	-0.047	-0.164	-0.484	-1.175
d)	0.033	0.095	0.220	0.447
e)	-0.104	-0.264	-0.589	-1.187
f)	0.041	0.137	0.385	0.962
Sum	-1.034	-2.971	-6.419	-11.375

Inclusion of Δ-isobars as 3N- and 4N-force



2N,3N, and 4N from
B.D.Day, PRC24,1203(81)

Rings with Δ-isobars :

Nucl. Phys. A389, 492 (1982)

PPNPhys 12, 529 (1983)

⇒ No sensible convergence with Δ-isobars

Pion-exchange channel dominates

- Decomposition in spin-isospin excitations

	S	M	T	Reid
third order	0	0	0	−0.302
	1	0	0	0.149
	1	1	0	0.059
	0	0	1	0.027
	1	0	1	−3.492
	1	1	1	0.540
sum				−3.019
fourth order	0	0	0	−0.060
	1	0	0	−0.017
	1	1	0	−0.012
	0	0	1	−0.004
	1	0	1	−0.755
	1	1	1	−0.317
sum				−1.166
total				−4.185

Nuclear Saturation without π-collectivity

- Variational calculations treat LRC (on average) and SRC simultaneously (Parquet equivalence) so **difficult** to separate LRC and SRC
- Remove 3-body ring diagram from Catania hole-line expansion calculation $\Rightarrow$ about the correct saturation density
- Hole-line expansion doesn't treat "real" Pauli principle very well
- Present results improve treatment of Pauli principle by self-consistency of spectral functions => more reasonable saturation density; binding energy?!?

- Neutron matter: pionic contributions must be included (Δ)

Pion collectivity: nuclei vs. nuclear matter

- Pion collectivity leads to pion condensation at higher density in nuclear matter (including Δ-isobars) => Migdal ...
- No such collectivity observed in nuclei $\Rightarrow$ LAMPF / Osaka data

- Momentum conservation in nuclear matter dramatically favors amplification of π-exchange interaction at fixed q
- In nuclei the same interaction is sampled over all momenta Phys. Lett. **B146**, 1(1984)

$$V_\pi(q) = -\frac{f_\pi^2}{m_\pi^2}\frac{q^2}{m_\pi^2 + q^2}$$

- Needs further study

$\Rightarrow$ Exclude collective pionic contributions to nuclear matter binding energy

Green's function V

Two Nuclear Matter Problems

The usual one

- With π-collectivity and only nucleons
- Variational + CBF and three hole-line results presumed OK (for E/A) but not directly relevant for comparison with nuclei!
- Add NNN —> adjust
- **NOT OK** if Δ-isobars are included explicitly
- Relevant for neutron matter

The relevant one?!

- Without π-collectivity
- Treat only SRC
- But at a sophisticated level by using self-consistency
- Confirmation from Ghent results $\Rightarrow$ Phys. Rev. Lett. **90**, 152501 (2003)
- 3N-forces difficult $\Rightarrow$ π ...

Some comparisons

- SNM

PHYSICAL REVIEW C **86**, 064001 (2012)

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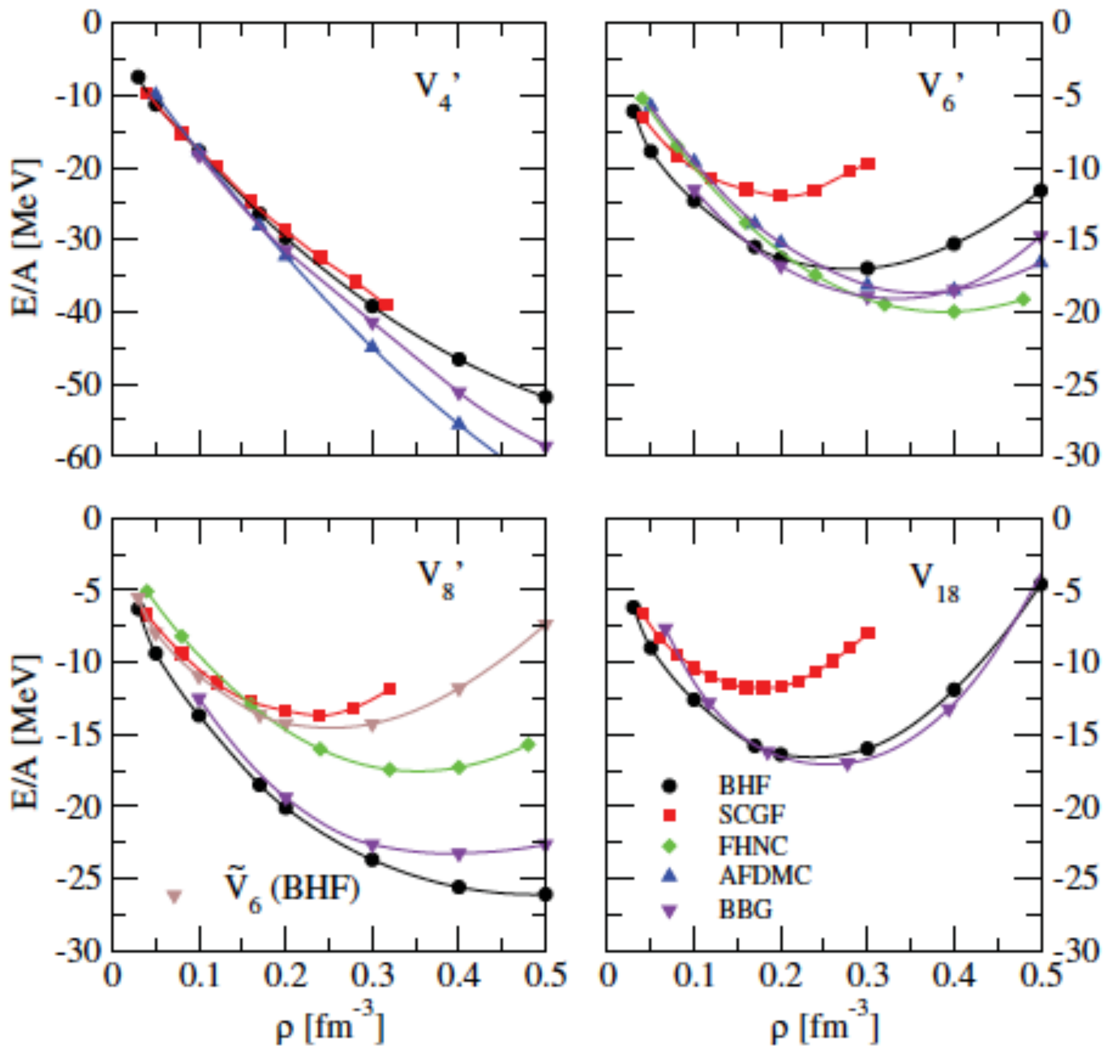


Have I changed my mind?

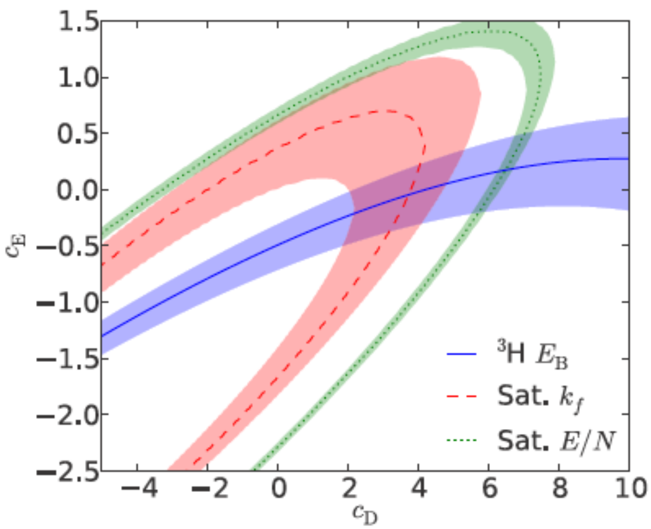
- Recent results for chiral interactions

- Systematic expansion in chiral perturbation theory
  - allows simultaneous construction of 2N and 3N interaction at appropriate orders
  - implemented with a very soft cut-off (500 MeV for example)
  - easy to compress nuclei —> small radii with NN
  - NNN strongly repulsive with higher density necessary

Nuclear matter saturation issues

- Old problem…
- Is it solved?
- Don't think so…

- Coupled cluster

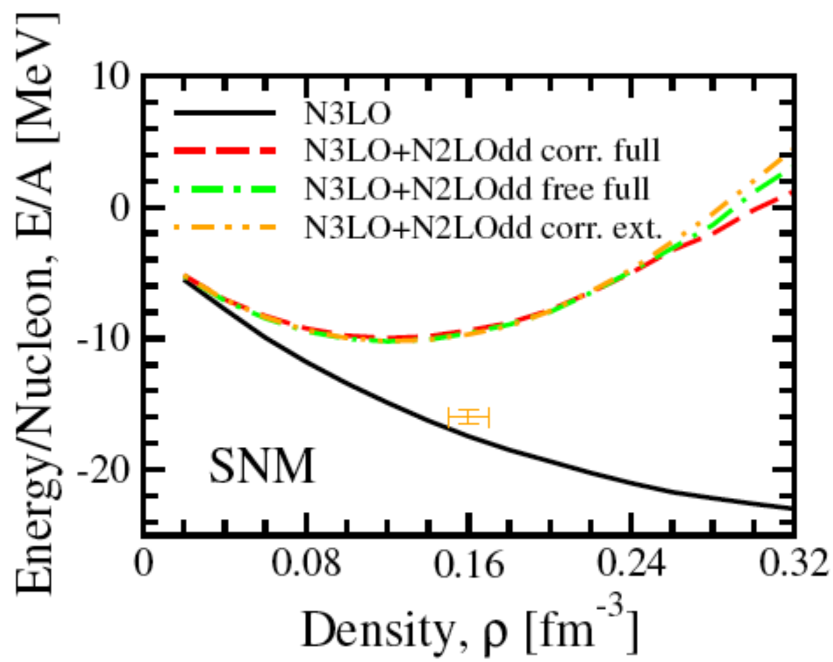


PRC **89**, 014319 (2014)
Can't do triton and saturation at the same time

- Lattice calculations

Radius of ^{16}O
$\langle r^2\rangle^{1/2}$=2.3 fm<—> Exp 2.71 fm
PRL112, 102501 (2014)

- SCGF only "SRC" no regulators



arXiv:1408.0717 PRC90,054322(2014)
3NF —> DD2NF

Saturation of symmetric nuclear matter: outlook

- Nuclear saturation problem
  - We know a lot …
  - We can't get it right …
  - Why not?
- Forces & methods
  - Chiral interactions + 3NF
    - Underbinds in SCGF (SRC only)
    - Coupled cluster: triton <-> nuclear matter cannot be reconciled
  - Comments
    - Not enough high-momentum content (JLab) —> NN interaction too soft
    - LRC (mainly pionic) contribute to energy
    - pion physics missing (NN static **only**???)
    - radii of heavier nuclei too small <—> saturation problem
    - empirical NNN in ^{40}Ca 1.28 MeV/A —> PRL 112, 162503 (2014)
- What to do?
  - Make chiral interactions consistent with JLab data (a little harder) —> good for finite nuclei as well
  - Continue to develop the techniques to deal with such a harder interaction (to be done for nuclei)
  - Revisit the formulation of the nuclear matter problem
    - Why?
      - pion-exchange in matter ≠ pion-exchange in a finite system
      - Liquid drop notion only good for very short-range physics
      - LRC normally small q —> no energy
      - Nuclear matter pions —> finite q —> increasing binding with density —> messes up saturation
      - see PRL90, 152501 (2003)