Lecture III-b: Renormalization group approaches for continuum couplings

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In the previous lecture...

What you have learned:

- Several many-body methods were successfully extended in the Berggren basis.

What you will learn (hopefully):

- Not all many-body basis are created equal to deal with continuum couplings.
- Renormalization group based techniques acknowledge the nature of the continuum.

Many-body methods in the Berggren basis

Configuration interaction based
(can be *ab initio*)

Gamow
shell model

↓

Factorial wall.

Gamow density matrix
renormalization group

Correlation truncated based
(*ab initio*)

Coupled clusters
in the Berggren basis

↓

Limited to closed-shell
nuclei ± 2 particles.

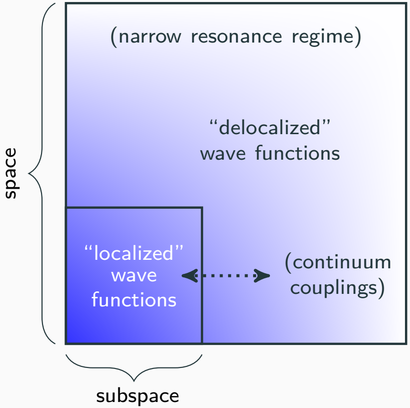
In-medium
similarity renormalization group
in the Berggren basis.

Renormalization group based

General observations about many-body methods in the Berggren basis

Open quantum systems definition:

→ Systems coupled to an environment of decay channels and scattering states.



Several strategies, one common point:

(▪ Feshbach projection formalism $\hat{Q} + \hat{P} = \hat{1}$ (CSM, SMEC).)

- Diagonalization in the Berggren basis (GSM).
- Factorization/Renormalization (G-DMRG).
- Similarity transformation/Renormalization (CC, IM-SRG).

→ Localized many-body wave function + "correction".

What changes is the way the correction (continuum couplings) is added.

RG methods acknowledge the nature of continuum couplings (natural division res./nonres. + perturbative correction).

Similarity transformations

General considerations:

- Similarity transformation $\hat{U}(s)$: $\hat{U}(s)\hat{U}^{-1}(s) = \hat{U}^{-1}(s)\hat{U}(s) = \hat{1}$.
- Similarity transformed Hamiltonian: $\hat{H}(s) = \hat{U}(s)\hat{H}\hat{U}^{-1}$.

Unitary if:
$\hat{U}^{-1}(s) = \hat{U}^{\dagger}$.

- The flow equation:

$$\begin{aligned}\frac{d\hat{H}(s)}{ds} &= \frac{d\hat{U}(s)}{ds}\hat{H}\hat{U}^{-1} + \hat{U}(s)\hat{H}\frac{d\hat{U}^{-1}(s)}{ds}\\ &= \frac{d\hat{U}(s)}{ds}\hat{U}^{-1}\hat{H} + \hat{H}\hat{U}(s)\frac{d\hat{U}^{-1}(s)}{ds}\\ &= \frac{d\hat{U}(s)}{ds}\hat{U}^{-1}\hat{H} - \hat{H}\frac{d\hat{U}(s)}{ds}\hat{U}^{-1}(s)\\ &= [\hat{\eta}(s), \hat{H}]_-\end{aligned}$$

- Flow parameter dependance:

$$\frac{d\hat{U}(s)}{ds}\hat{U}^{-1}(s) + \hat{U}(s)\frac{d\hat{U}^{-1}(s)}{ds} = 0.$$

$$\Rightarrow \hat{\eta}(s) = -\hat{\eta}^{-1}(s).$$

- Flow generator: $\hat{\eta}(s) = \dfrac{d\hat{U}(s)}{ds}\hat{U}^{-1}(s)$.

$\rightarrow$ One needs to know either $\hat{U}(s)$ or $\hat{\eta}(s)$.

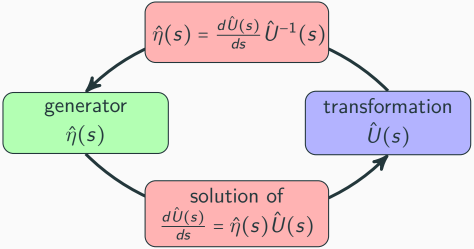
Similarity transformations

General considerations:

- Non-Hermitian Hamiltonian: $\hat{H} = \frac{1}{2}(\hat{H} + \hat{H}^\dagger) + \frac{1}{2}(\hat{H} - \hat{H}^\dagger) = \hat{H}_h + \hat{H}_{ah}$.
- Wegner generator: $\hat{\eta}(s) = [\hat{H}_d, \hat{H}_{od}]_-$.
- The condition for unitarity is $\hat{\eta}^\dagger(s) = -\hat{\eta}(s)$, but in fact: $\hat{\eta}^\dagger(s) = -[\hat{H}_{\mathrm{h,d}}, \hat{H}_{\mathrm{h,od}} - \hat{H}_{\mathrm{ah,od}}]_- \neq -\hat{\eta}(s)$.

$\rightarrow$ Non-Hermitian Hamiltonians cannot be transformed unitarily!

- Coupled clusters: $\hat{U}_{CC}(s) = e^{\hat{T}(s)}$ is known, $\hat{\eta}_{CC}(s)$ is not.
- IM-SRG: $\hat{\eta}(s)$ is known (or chosen), $\hat{U}(s)$ is not (it could).



Connections between CC and IM-SRG

The CC flow generator:

$$\hat{\eta}_{CC}(s) = \frac{d\hat{U}(s)}{ds}\hat{U}^{-1}(s) = \frac{d(e^{\hat{T}(s)})}{ds}e^{-\hat{T}(s)}$$
$$= \frac{d\hat{T}(s)}{ds} + \frac{1}{2!}\left[\hat{T}(s), \frac{d\hat{T}(s)}{ds}\right]_- + \frac{1}{3!}\left[\hat{T}(s), \left[\hat{T}(s), \frac{d\hat{T}(s)}{ds}\right]_-\right]_- + \cdots$$

→ No closed form can be inferred after the Magnus expansion...

Magnus expansion:

$\hat{U}(s) = e^{\hat{\Omega}(s)}$, $\hat{\Omega}^\dagger(s) = -\hat{\Omega}(s)$, $\hat{\Omega}(0) = 0$.

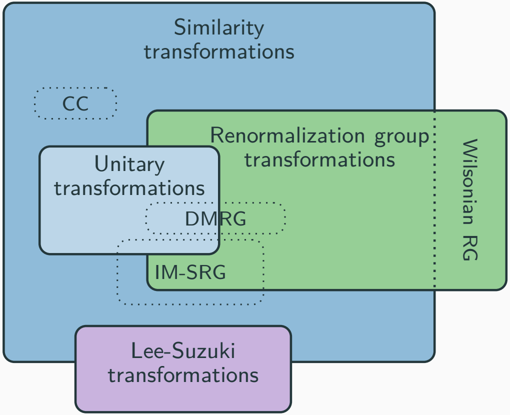
- Hamiltonian:
  $$\hat{H}(s) = e^{\hat{\Omega}(s)}\hat{H}e^{-\hat{\Omega}(s)} = \sum_{k=0}^{\infty}\frac{1}{k!}\mathrm{ad}^k_{\hat{\Omega}(s)}(\hat{H})$$
  with $\mathrm{ad}^0_{\hat{\Omega}(s)}(\hat{\eta}(s)) = \hat{\eta}(s)$ and
  $$\mathrm{ad}^k_{\hat{\Omega}(s)}(\hat{\eta}(s)) = \left[\hat{\Omega}(s), \mathrm{ad}^{k-1}_{\hat{\Omega}(s)}(\hat{\eta}(s))\right]_-.$$
- Derivative:
  $$\frac{d\hat{\Omega}(s)}{ds} = \sum_{k=0}^{\infty}\frac{B_k}{k!}\mathrm{ad}^k_{\hat{\Omega}(s)}(\hat{\eta}(s))$$

Guessing $\hat{\eta}_{CC}(s)$ from IM-SRG?

$$\hat{\eta}_{\text{White, MP}}(s) = \sum_{p,h}\frac{f_{ph}(s)}{f_p - f_h}\{\hat{a}^\dagger_p\hat{a}_h\} + \sum_{p,p',h,h'}\frac{\Gamma_{pp'hh'}(s)}{f_p + f_{p'} - f_h - f_{h'}}\{\hat{a}^\dagger_p\hat{a}^\dagger_{p'}\hat{a}_{h'}\hat{a}_h\} - \text{H.c.} \quad \Rightarrow \quad \hat{U}(s) = ?$$

Connections with RG

Renormalization group in perspective:



- Basic idea:
  N d.o.f. $\rightarrow N'$ new d.o.f. with $N' < N$.
- CC could be based on a unitary transformation, but the CC expansion would not self-truncate.
- IM-SRG does not a proper RG transformation with the White flow generator.
- DMRG is by definition an RG transformation (unitary or not).

Thank you for your attention!